Let's briefly revisit overall with physical meaning of Maxwell's equations:

electro-magnetic fields in vacuum in any inertial point P and time t will be determined by force vector: $\vec{F}(t,P)$ $\vec{F}(t,P)$. We can simply write the force that experiment point charge q moves with speed $\vec{v}$: $\vec{F} = q\vec{E} + q[\vec{v}\times\vec{B}]$ — this is Lorentz force.

What Lorentz force tells us:

① Electric field can be found through the force that experiment point charge when it is static.

② Magnetic field $\vec{B}(t,P)$ is only defined the part of $\vec{F}$ that appears when the charge is moving.

What can tere we the source of electro-magnetic radiation?

① ρ - density of charge

② $\vec{j}$ - vector of current density.

Maxwell's equations:

$$\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \operatorname{div}\vec{B} = 0$$

$$c\times\operatorname{rot}\vec{B} = \frac{\partial\vec{E}}{\partial t} + \frac{\vec{j}}{\varepsilon_0} \qquad \operatorname{rot}\vec{E} = -\frac{\partial\vec{B}}{\partial t} = 0$$

Here c is electro-magnetic constant that is

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

Little math refreshment

① $\operatorname{div}\vec{a}$? Let us write $\vec{a}(x,y,z) = \vec{a}_x\vec{e}_x + \vec{a}_y\vec{e}_y + \vec{a}_z\vec{e}_z$

$$\operatorname{div}\vec{a} = (\vec{\nabla}\vec{a}) = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

Meaning:

- divergence is the measure of vector field's increase in the direction its positive
- measure of the field's tendency to diverge from that point

② $\operatorname{rot}\vec{a}$?

$$\operatorname{rot}\vec{a} = [\vec{\nabla}\times\vec{a}] =$$

$$= \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right)\vec{e}_x + \left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}\right)\vec{e}_y$$

$$- \left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)\vec{e}_z$$

$$\operatorname{rot}\vec{a} = [\vec{\nabla}\times\vec{a}] = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix}$$

$$\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0}$$

Let's take volume V, will closed surface S:

$$\int_V \operatorname{div}\vec{E}\,dV = \frac{1}{\varepsilon_0}\int_V \rho\,dV$$

Mathematical: Ostrogradsky - Gauss theorem

$$\oint_S \vec{E}\,d\vec{S} = \frac{1}{\varepsilon_0}\int_V \rho\,dV$$

(1.1) — Gauss law

What Gauss law tells us:

The flow of electric field through closed surface S is determined by the total charge that is confined in the closed volume V.

It will be enough to demonstrate that in case of static, not moving charges it is Coulomb law.

Strictly mathematically speaking it is not possible as we do have law information on $\operatorname{rot}\vec{E}$. However, if we assume electrostatic point charge

$$\oint_S \vec{E}\,d\vec{S} = \frac{1}{\varepsilon_0}\int_V \rho\,dV = \frac{Q}{\varepsilon_0}$$

$$\text{with } r(\vec{r}\,\vec{E}) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \longrightarrow \text{Coulomb law}$$

what about dimensionality?

$$c^2\times\operatorname{rot}\vec{B} = \frac{\partial\vec{E}}{\partial t} + \frac{\vec{j}}{\varepsilon_0}$$

Now we take closed contour ℓ and open surface S that is supported by ℓ.

Let's calculate the flow of (1.2) left and right parts through surface S

$$\int_S (c^2\operatorname{rot}\vec{B} - \frac{\partial\vec{E}}{\partial t})\,d\vec{S} = \frac{1}{\varepsilon_0}\int_S \vec{j}\,d\vec{S}$$

Maxwell-Stokes theorem

$$c^2\oint_\ell \vec{B}\,d\vec{l} = \frac{1}{\varepsilon_0}\int_S \vec{j}\,d\vec{S} - \frac{\partial}{\partial t}\int_S \vec{E}\,d\vec{S}$$

What such law means:

1) Circulation of vector $\vec{B}$ on any closed contour is determined by total current I through the surface S.

Ampere's law (Biot-Savart law)

2) It means the changing electric field $\vec{E}$ ($\frac{\partial\vec{E}}{\partial t}$) creates vortex magnetic field.

$$\vec{B} \neq \vec{E}(t)$$

Maxwell brought up the new term - Displacement current $\frac{\partial\vec{E}}{\partial t}$. Why?

This appeared to be a crucial point to bring unified theory of $\begin{cases}\text{Electric}\\ \text{Magnetic}\\ \text{Optics}\end{cases}$

This were historically two types of current:

1) conduction current $\vec{j}$

This is directional movement of electric charges on long distances

$$\vec{a} \longrightarrow$$

We can easily measure it in closed circuit by ampere-meter

2) So-called current from bound charges: the average displacement of charges (electrons, nucleus) compared to nucleus center.

The main argument was to explain possibility of registration of current for open circuit if alternative current is used.

Changing $\frac{\partial\vec{E}}{\partial t}$ Maxwell made into that the charge conservation law has been fulfilled

Let's show it.

We know (1.2)

$$c^2\operatorname{rot}\vec{B} = \frac{\partial\vec{E}}{\partial t} + \frac{1}{\varepsilon_0}\vec{j}$$

Let's see how it diverges from initial point, i.e.

$$\operatorname{div}(c^2\operatorname{rot}\vec{B}) = \frac{\partial}{\partial t}\operatorname{div}\vec{E} + \frac{1}{\varepsilon_0}\operatorname{div}\vec{j}$$

$$= \frac{\partial}{\partial t}\operatorname{div}\vec{E} + \frac{1}{\varepsilon_0}\operatorname{div}\vec{j}$$

Since $\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0}$

$$= \frac{\partial\rho}{\partial t} + \operatorname{div}\vec{j}$$

$$\frac{\partial}{\partial t}\int_V \rho\,dV = -\int_V \operatorname{div}\vec{j}\,dV \longrightarrow \text{Charge conservation law}$$

The change of the total charge in closed volume V can be only done by carrying it through the surface S, that defines this volume.

$$\operatorname{rot}\vec{E} = -\frac{\partial\vec{B}}{\partial t} \neq 0$$

$$\int_S \operatorname{rot}\vec{E}\,d\vec{S} = -\int_S \frac{\partial\vec{B}}{\partial t}\,d\vec{S} = 0$$

$$\oint_\ell \vec{E}\,d\vec{l} = -\frac{\partial}{\partial t}\int_S \vec{B}\,d\vec{S} \longrightarrow \text{Faraday law}$$

Faraday experimentally demonstrated that electromotive force (EMF) is defined by the speed of magnetic flow change through surface S.

$$\underbrace{\oint_\ell \vec{E}\,d\vec{l}}_{\text{EMF}} = -\frac{\partial}{\partial t}\underbrace{\int_S \vec{B}\,d\vec{S}}_{\text{flux}}$$

speed

EMF is defined as electromagnetic work done on a unit charge when it has travelled one full round in closed loop

$$\frac{\partial B}{\partial t}$$

In other words, varying magnetic fields lead to varying electric fields and vice versa.

$$\operatorname{div}\vec{B} = 0$$

$$\oint_S \vec{B}\,d\vec{S} = 0$$

The flow of magnetic field through any closed surface S is 0

What does it means?

It means that $\vec{B}$ is closed contour

Gauss law of magnetism

Finally, Maxwell's equations

$$\operatorname{div}\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\operatorname{div}\vec{B} = 0$$

$$c^2\operatorname{rot}\vec{B} = \frac{\partial\vec{E}}{\partial t} + \frac{1}{\varepsilon_0}\vec{j}$$

$$\operatorname{rot}\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

We will add so-called mechanical equation:

$$\vec{D} = \varepsilon\varepsilon_0\vec{E} \quad \text{for properties}$$